AFRL-SR-AR-TR-10-0207

REPORT DOCUMENTATION PAGE

1. REPORT DATE *(DD-MM-YYYY)* 16 MAY 09	2. REPORT TYPE FINAL REPORT	3. DATES COVERED *(From - To)* 01 APR 07 TO 30 NOV 09
4. TITLE AND SUBTITLE PLANNING UNDER UNCERTAINTY: METHODS AND APPLICATIONS		5a. CONTRACT NUMBER FA9550-07-1-0389
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) PROF STEPHEN ROBINSON		5d. PROJECT NUMBER 2304
		5e. TASK NUMBER DX
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) UNIVERSITY OF WISCONSIN SYSTEM, RESEARCH & SPONSORED PROGRAMS, 21 N PARK ST STE 6401 MADISON, WI 53715-1218		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NL 875 NORTH RANDOLPH STREET SUITE 325, ROOM 3112 ARLINGTON, VA 2203-1768		10. SPONSOR/MONITOR'S ACRONYM(S)
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE. DISTRIBUTION IS UNLIMITED

13. SUPPLEMENTARY NOTES

14. ABSTRACT

This ambitious projects that would enhance both the optimization tools and modeling, and increasing computing power based on rigorous mathematical analysis, can have far-reaching impact on large-scale optimization and simulation problems of interest to the Air Force such as military logistics and planning.

15. SUBJECT TERMS

20100617297

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					19b. TELEPHONE NUMBER *(Include area code)*

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

DTIC Data

Purchase Request Number:	FQ8671-0900025
BPN:	F1ATA08224B021
Proposal Number:	06-NM-302
Research Title:	PLANNING UNDER UNCERTAINTY: METHODS AND APPLICATIONS
Type Submission:	Final Report
Inst. Control Number:	FA9550-07-1-0389P00002
Institution:	BOARD OF REGENTS OF THE UNIV OF WISCONSIN SYSTEM
Primary Investigator:	Professor Stephen M. Robinson
Invention Ind:	none
Project/Task:	2304D / X
Program Manager:	Donald Hearn

Objective:

This objectives of this research are three-fold: Developing better planning tools, increasing computer power by novel techniques for making better use of computing tools, and enhancing modeling paradigms by using variational inequalities, and composition duality.

Approach:

In order to achieve the first goal of developing better planning tools, the PIs plan to work on investigating further classes of Stochastic Mixed Integer Programming (SMIP) in problems where optimization of constraints of uncertain variables subject to constraints is required. Simulation-based optimization that integrates optimization techniques into the simulation analysis will also be considered by used of surrogate models which would reduce the computational complexity and cost of the simulations. For increasing computing power for optimization, grid computing approaches, decomposition of problem space in to sub-problems, partitioning of feasible region, integration of decomposition and meta heuristics and nonlinear preprocessing will be considered. The last goal on enhancing modeling paradigms will be achieved by considering innovative approaches such as analysizing structural properties of variational conditions, solvers for variational inequalities, and composition duality, which is a transformation technique to reformulate the original large-scale problems in an equivalent but smaller dimension dual space.

Progress:

Year: 2007 **Month:** 01

Not required at this time.

Year: 2008 **Month:** 04

We have made progress in several areas of computational optimization, including optimization of simulations (with a toolbox under development for general deployment), stochastic programming methods for dose shaping in radiotherapy, crop management schemes for sustainable agriculture, large?scale complementarity problems, and stochastic equilibria. We have also developed new reparametrization methods that greatly simplify the analysis of variational conditions. Furthermore, we have continued our work in the analysis of basic mixed integer programming models that frequently appear in production planning under uncertainty. We have defined results that enable us not only to solve these models, but also more complicated problems that contain them as submodels. We have also begun applying mixed integer programming methods to basic problems in health care delivery that involve both data uncertainty (e.g., uncertain surgery durations) and combinatorial components (e.g., fixed costs, indivisible assignment of resources, etc.). Finally, we have begun research into fundamental algorithms for optimization and re?optimization of continuous optimization problems (such as linear and quadratic programs). These methods, which take into account the underlying polyhedral structure

Progress:

Year: 2008 **Month:** 04

without requiring that only extreme point solutions be considered, have numerous desirable properties related to improving directions, degeneracy resolution, avoidance of pathological pitfalls, and asymptotic complexity.

Year: 2009 **Month:** 03

Major accomplishments during the past year:

1. Transmission switching within the design of smart electrical grids, showing system savings both on benchmark networks, such as the IEEE 118 bus system, and large-scale networks, including the ISO-NE systems. Demonstrated savings of 13% to 25% (with Oren (Berkeley) and O'Neill (FERC)).

2. Optimization of large scale mixed integer programming models arising in process design, power systems and coal mining (with Maravelias (Madison).

3. Simulation optimization in the design of a robust sleeve antenna for hepatic microwave ablation. Our Bayesian VNSP algorithm yields a 14.3% improvement over the original design while saving 68.2 % of the simulation evaluations compared to standard sample-path optimization.

4. Simulation optimization for the risk averse second best toll pricing problem. Existing methods may produce risk-prone designs that try to optimize for the best-case scenario. We have developed a risk-averse approach and demonstrated conditions for the existence of a solution.

5. Local analysis of equilibrium-type problems including variationalinequalities. We have shown how to do sensitivity analysis of equations on the graph of a maximal monotone operator. These include variational inequalities and many other problems such asRockafellar's extended nonlinear programming. They provide tools for building and justifying computational algorithms for such problems.

Year: 2010 **Month:** 03 **Final**

Research under this grant commenced 01 April 2007 and the grant continued through 30 Nov 2009, continuing the program formerly funded under Grant FA9550-04-1-0192. Progress was achieved in a broad range of areas, including (1) developing better planning tools; (2) providing increased computing power for solving planning problems; (3) creating enhanced modeling paradigms. Section 2 describes this progress in more detail. This work was described and documented in a total of one book and 22 research papers, as well as doctoral theses. Section 4 lists these publications with their status. Section 5 lists scientific interactions as well as consultative and advisory service to DoD agencies.

FINAL REPORT

AFOSR Grant FA9550-07-1-0389

Planning Under Uncertainty: Methods and Applications

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Period covered: 01 April 2007 - 30 November 2009

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Contents

1 Objectives

The objectives of this research did not change from the original proposal, where they were summarized as follows.

> The proposers plan to pursue a program of research over a 36-month period with the following goals and objectives:
>
> ## 1.1 Goal: Develop better planning tools
>
> Employ structural features of stochastic mixed integer programming problems to develop and test enhanced solution methods.
>
> Develop and test improved methods for simulation optimization (that is, for adjusting parameters of a model to optimize a performance function that can be observed only by simulation).
>
> Apply the resulting methods to problems including scheduling and planning in health care.
>
> ## 1.2 Goal: Provide increased computing power for solving planning problems
>
> Develop and test solution techniques combining decomposition methods with metaheuristics, with the aim of producing highly effective solution methods.
>
> Devise and test grid computing methods that coordinate the simultaneous efforts of many computers to solve large and/or difficult problems.
>
> Develop preprocessors with the aim of greatly enhancing the efficiency of solution methods for optimization and equilibrium problems.
>
> Test the resulting methods on planning and resource allocation problems from various application areas.
>
> ## 1.3 Goal: Create enhanced modeling paradigms
>
> Improve current modeling languages and solvers to permit users to specify directly, and solve quickly, complex equilibrium problems modeled by variational inequalities.

Devise and justify new solution algorithms for variational inequality problems based upon a comprehensive investigation of transformation methods including composition duality.

Test the resulting methods on planning and resource allocation problems from various application areas.

We made progress in each of the three application areas as well as in the area of theoretical foundations and computational tools. This progress is reported in the narrative of Section 2 as well as in the publication items, submitted and accepted, listed in Section 4.

2 Accomplishments/new findings

The following sections describe research highlights, significance, relevance, and potential applications. They are organized according to the three main application areas described in Section 1, together with the supporting area of theoretical foundations and computational tools.

2.1 Develop better planning tools

- We have investigated the notion of transmission switching within the design of smart grid [17, 16, 21], including aspects of capacity expansion for the electrical power grid in collaboration with Oren (Berkeley) and O'Neill (FERC). Bulk electricity networks are made up of generators that produce power, loads that consume power, and transmission lines that transmit power from generators to loads. Electricity networks are typically geographically vast and have complex interconnections, with thousands of lines connected in a web-like pattern under the control of a single network operator. In a network, generators are dispatched, or operated, in such a way that they meet the power demand of the loads but still respect power flow limits on each transmission line. Power flows from generators to loads along the lines in these networks in accordance with the laws of physics, which dictate that power flow along all lines in proportion to the electrical characteristics of those lines. This means the flow on each line is influenced by the properties of all other lines in the network. Therefore, it is possible to remove a line and improve the throughput of the system.

Optimal transmission switching is an innovative electricity grid management paradigm that takes advantage of this property of electricity networks. Transmission lines are switched in and out of the network in order to maximize economic efficiency of generation dispatch on a bulk electricity network. The optimal transmission switching problem can be formulated as an optimization problem with binary variables representing the status of each line. Using this formulation to dispatch generators and determine the status of each line, our team has demonstrated system savings both on benchmark networks, such as the IEEE 118 bus system, and large-scale networks, including the ISO-NE systems. Savings found here range from 13% to 25%, which, in a multi-trillion dollar industry, can mean substantial savings. In addition, market settlements based on OPF with transmission switching are examined and discussed. We have also formulated a security-constrained OPF, and explored the savings possible with optimal transmission switching in this more secure (and necessarily more restrictive) problem.

- We have continued to investigate the optimization of simulations [7, 8, 29, 9]. Specifically, we are interested in developing techniques to optimize black box simulation codes. We have considered two application areas for this work, the first being the design optimization of a robust sleeve antenna for hepatic microwave ablation. We utilize a Bayesian variable-number sample-path (VNSP) optimization algorithm to yield a robust design for a floating sleeve antenna for hepatic microwave ablation. Finite element models are used to generate the electromagnetic (EM) field and thermal distribution in liver given a particular design. Dielectric properties of the tissue are assumed to vary within $\pm$ 10% of average properties to simulate the variation among individuals. The Bayesian VNSP algorithm yields an optimal design that is a 14.3% improvement over the original design and is more robust in terms of lesion size, shape and efficiency. Moreover, the Bayesian VNSP algorithm finds an optimal solution saving 68.2% of the simulation evaluations compared to the standard sample-path optimization method.

  We have also investigated joining this work with our approaches for solving complementarity systems by considering the risk averse second best toll pricing problem. Existing second best toll pricing (SBTP) models determine optimal tolls of a subset of links in a transportation network by minimizing a certain system objective, while the traffic flow pattern is assumed to follow user equilibrium (UE). We show in our paper that such toll

design approach is risk prone, which tries to optimize for the best-case scenario, if the UE problem has multiple solutions. Accordingly, we propose a risk averse SBTP approach aiming to optimize for the worst-case scenario, which can be formulated as a min-max problem. We establish a general solution existence condition for the risk averse model and discuss in detail that such a condition may not be always satisfied in reality. In case a solution does not exist, it is possible to replace the exact UE solution set by a set of approximate solutions. This replacement guarantees the solution existence of the risk averse model. We then develop a scheme such that the solution set of an affine UE can be explicitly expressed.

- We have considered the approximation of a given $m \times n$ nonnegative real matrix A by a weighted nonnegative sum of binary matrices. Such approximation problems arise in Intensity Modulated Arc Therapy (IMAT), an advanced form of radiotherapy for cancer [20]. In that context, the binary matrices S_i represent the "open" positions for the radiation beamlets in a so-called aperture, and the corresponding weights denote the radiation intensity associated with the aperture. The weighted sum of apertures thus represents an approximation (typically based on a small number of apertures) to the ideal radiation distribution at a given angle given by the matrix A. The determination of an optimal approximation of this type for a limited number of S_i's is termed a segmentation problem. The approximation problem considered in our IMAT work is a set of coupled segmentation problems with delivery constraints. The delivery constraints are mechanical and limit both the types of apertures that may be employed as well as the changes that may be made to apertures between successive angles as the radiation delivery equipment moves in sweeps along an arc around the patient. We present an effective heuristic algorithm using network optimization models to solve this IMAT optimization problem.

- We have developed polynomial-time algorithms for uncapacitated lot-sizing problems under uncertainty [19], and also for such problems with backlogging [18].

- We have developed stochastic optimization models for the allocation of surgery procedures to operating rooms in the presence of uncertainty about the duration of the procedures. These included a two-stage stochastic linear-programming problem, the robust counterpart of that problem, and a fast and easy-to-implement heuristic [10].

2.2 Provide increased computing power for solving planning problems

- We have considered the optimization of large scale mixed integer programming models arising in process design, power systems and coal mining [14, 13]. Scheduling problems arise in many applications in process industries. However, despite various efforts to develop efficient scheduling methods, current approaches cannot be used to solve instances of industrial importance in reasonable time frames. Our work develops a dynamic decomposition scheme that exploits the structure of the problem and is well suited for grid computing The problem we study is the simultaneous batching and scheduling of multi-stage batch processes. The algorithm first decomposes the original problem into 1st-level subproblems based on batching decisions. The subproblems that remain unsolved within a resource limit are then dynamically decomposed into 2nd-level subproblems based on batch-unit assignment decisions in one stage. The process can be repeated in a dynamic fashion by identifying subproblems that cannot be solved within a given resource limit and decomposing them by batch-unit assignment, until all subproblems are solved. Alternatively, a problem can be decomposed into a number of promising subproblems using an automatic strong branching scheme. Our results show that the proposed method can be used on a grid computer to solve large problems to optimality in reasonable computational time.

- Our paper [6] describes a framework for modeling optimization problems for solution on a grid computer. The framework is easy to adapt to multiple grid engines, and can seamlessly integrate evolving mechanisms from particular computing platforms. It facilitates the widely used master/worker model of computing and is shown to be flexible and powerful enough for a large variety of optimization applications. In particular, we summarize a number of new features of the GAMS modeling system that provide a lightweight, portable and powerful framework for optimization on a grid. We provide downloadable examples of its use in several financial applications with clear strategies for parallel solution, for decomposition and iterative algorithms, and for solving very difficult mixed integer programs to optimality. Computational results are provided for a number of different grid engines, including multi-core machines, a pool of machines controlled by the Condor resource manager and the grid engine from Sun Microsys-

tems.

2.3 Create enhanced modeling paradigms

- We have continued to explore the use of Mathematical Programs with Equilibrium Constraints (MPEC's) in the context of traffic infrastructure design and tolling mechanisms [3, 3, 1, 2, 5, 4].

  The Second-Best Toll Pricing (SBTP) problem aims to determine an optimal toll scheme for a given set of links in a transportation network so that traffic can be distributed more efficiently from the system point of view. SBTP can be further categorized as static SBTP or dynamic SBTP (DSBTP), depending on whether traffic dynamics are considered or not. We have focussed on the static SBTP and have modeled SBTP as a bilevel problem or an MPEC. Specific design of algorithms for the solution of these problems have been investigated.

  We have also formulated the continuous network design problem under asymmetric user equilibrium as a mathematical program with complementarity constraints, with the upper level a nonlinear programming problem and the lower level a nonlinear complementarity problem. Unlike most previous studies, the proposed framework is more general, in which both symmetric and asymmetric user equilibrium can be captured. By applying the complementarity slackness condition of the lower level problem, the original bilevel formulation can be converted into a single level and smooth nonlinear programming problem. In order to solve the problem, a relaxation scheme is applied by progressively restricting the complementarity condition, which has been proven to be a rigorous approach under certain conditions. The model and solution algorithm are tested for well-known network design problems and promising results are given.

- Extended mathematical programs are collections of functions and variables joined together using specific optimization and complementarity primitives [11, 12]. We have outlined a mechanism to describe such an extended mathematical program by means of annotating the existing relationships within a model to facilitate higher level structure identification. The structures, which often involve constraints on the solution sets of other models or complementarity relationships, can be exploited by modern large scale mathematical programming algorithms for efficient solution. Example applica-

tions are taken from chemical engineering and operations research.

A number of new modeling formats involving complementarity and variational inequalities have been designed and a framework, EMP, that allows such problems to be specified has been implemented. We believe this will make a modeler's task easier by allowing model structure to be described succinctly. Furthermore, model generation can be done more reliably and automatically, and algorithms can exploit model structure to improve solution speed and robustness.

We believe that EMP is useful in systems optimization where collections of interacting (optimization and complementarity) models need to be processed. The automatic reformulations of these problems will save time, improve accuracy, and expand the range of problems that can be practicably solved. We have already seen the importance of these developments in several application areas, including work on stochastic equilibria where multiple agents are solving optimization problems under a governing equilibrium principle, and in bilevel problems arising in bio-engineering. While the implementation described in our work was developed for the GAMS modeling system, we believe that most of the features are applicable to any modeling system. Specifically, equation annotations can be provided in AMPL via user-defined constructs while the object oriented design of MATLAB could easily be extended to this setting.

In conjunction with the thesis of student Li, we have developed efficient implementations and investigated new algorithms to robustly solve variational models over general polyhedral convex sets and perform sensitivity analyses. The methodology utilizes large scale numerical linear algebra techniques that have been developed for use with the PATH solver [22]. While the underlying pivotal mechanisms are similar (involving factor, solve and updates) we show how to use modern linear algebra packages in the particular context of solving complementarity and variational inequality systems.

- Other work has been carried out on the theory and algorithms for optimization. Specifically, we have investigated the linear algebra needed for solving classes of visualization problems with an underlying complementarity model of friction [15].

  We have looked at the notion of the uniqueness of integer solutions to systems of linear equations and have characterized properties of the underlying system to allow identification of this property [27].

We have also considered the use of global optimization techniques for the design of electrical motors and have built a number of small scale but highly nonlinear models that facilitate understanding of the tradeoffs in designs [28].

2.4 Theoretical foundations and computational tools

We investigated several foundational and computational areas with the aim of improving the understanding of solution behavior and the computation of solutions, for optimization problems as well as for variational conditions. The following subsections describe findings and accomplishments in these areas.

2.4.1 Variational inequalities with perturbed polyhedral constraints

Although the behavior of solutions for nonlinear variational inequalities with (fixed) polyhedral constraint sets is well understood, the corresponding behavior for problems whose constraint sets are subject to perturbation has not been. We have obtained a relatively complete solution to the latter problem in the case for which the perturbations occur in the right-hand sides of the linear equations and inequalities defining the constraint set. Conditions that are sufficient and, under additional hypotheses, also necessary for local uniqueness and Lipschitz stability appear in [26], with supplementary analysis in [25], both sponsored by this grant. This work was applied to the sensitivity analysis of fixed-demand traffic equilibrium problems in [24].

A separate question for the same class of problems involves the situation in which the solution is not required to be locally unique, but in which the function involved in the variational inequality is required to be affine: that is, the problem is of the form

$$0 \in Ax + a + N_{C(s)}(x), \tag{1}$$

where $C(s) = \{x \mid Rx + Ts \leq t\}$, so that the parameter s controls perturbations in $C(s)$. For this problem we were able to show that if the matrix A is positive semidefinite and *cocoercive* (that is, if $\langle x, Ax \rangle = 0$ then $Ax = 0$; this is also called the *Dunn property*), then if (1) has a solution, the solution set is Lipschitz continuous in the Hausdorff metric when considered as a function of s on the set of s for which there is a solution. Further, we showed that the cocoercive matrices were the largest class of positive semidefinite matrices for which such a result could hold. These results, given in [34], made it possible to extend known results about

good behavior in quadratic programming problems to the more general case of (1), in which the matrix A need not be symmetric.

2.4.2 Analysis of equations on monotone graphs

A mathematical structure that has proven to be pervasive in multiple areas of applied mathematics, including operations research, is the *variational inequality.* Given a (generally closed) convex set $C \subset \mathbb{R}^n$ and a function $f : C \longrightarrow \mathbb{R}^n$, a point x_0 satisfies the variational inequality (VI) defined by the pair (f, C) if

$$0 \in f(x_0) + N_C(x_0), \tag{2}$$

where N_C is the *normal-cone operator* of C, defined by

$$N_C(x) = \begin{cases} \{x^* \mid \text{For each } c \in C,\ \langle x^*, c - x\rangle \leq 0\} & \text{if } x \in C, \\ \emptyset & \text{if } x \notin C. \end{cases}$$

Models expressible by VI include optimality conditions for nonlinear-programming problems, complementarity problems (in which $C = \mathbb{R}^n_+$), traffic equilibrium problems, free boundary problems in physical mathematics, economic equilibrium formulations in production, and many others. Accordingly, it is of considerable importance to know conditions under which VI will have solutions and, if they do, then conditions under which small changes in the data of the VI will result in only small changes in the solution. This latter property means that the problem expressed by the VI is *well posed,* and it is generally a prerequisite for the use of such a VI to model a practical problem.

If we define the *graph* of a multifunction $M : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ to be the set of pairs $\operatorname{gph} M := \{(x, x^*) \mid x^* \in F(x)\}$, then the expression $0 \in f(x) + N_C(x)$ is equivalent to

$$f(x) + x^* = 0, \qquad (x, x^*) \in \operatorname{gph} N_C. \tag{3}$$

The operator N_C is known to be maximal monotone, so that (3) contains an equation defined on the graph of the monotone operator N_C.

In (3) the equation involves a function of (x, x^*) of very special form: namely, $f(x) + x^*$. But there is no reason why we need to restrict ourselves to functions of that form; instead we can consider a maximal monotone operator $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and a function $g : \operatorname{gph} T \longrightarrow \mathbb{R}^n$, and write the more general problem

$$g(x, x^*) = 0, \qquad (x, x^*) \in \operatorname{gph} T. \tag{4}$$

The problem (4) then belongs to the category of *equations on monotone graphs (EMG)*. Since such equations include variational inequalities as a special case, they include a very large class of problems of practical importance. In addition, the special form (3), in the case when C is polyhedral, leads to the normal-map formulation that we have investigated under prior Air Force support [30, 31, 33]. This has proven to be effective for computational treatment of variational inequalities.

We have been able to establish an implicit-function theorem for problems of the form (4), for problems in which g is C^1 and a certain injectivity condition holds. When T is the normal-cone operator of a polyhedral convex set C, then the required condition is a generalization of the well-known condition for tractability of the ordinary nonlinear equation $h(x) = 0$, where $h : \mathbb{R}^n \to \mathbb{R}^n$ is C^1, which is that the derivative of h at the point under investigation be nonsingular. This work was written up and submitted for publication; however, additional work subsequently led to a radically shorter and simpler proof of the implicit-function theorem. Accordingly, the paper was withdrawn from the publication process for revision and is being rewritten to incorporate the shorter proof. In the meantime, these results have been put to work in the development of computational methods, as described in the following section.

2.4.3 Newton's method for equations on monotone graphs

Various authors have shown how to extend the Newton method to the case of equations that are nonsmooth, so that conventional derivatives do not exist at some points. In particular, in [32] a theorem of Kantorovich type was established for the class of problems admitting *point-based approximations,* which are functions that generalize the linearizations employed in the conventional Newton method.

With support from this grant we have shown that if the generalized nonsingularity condition mentioned in the previous section holds at a solution of an EMG, then provided that one starts the Newton process sufficiently close to that solution the process will converge R-quadratically to it. This extends to EMG the classical result that if at a solution of an ordinary equation the derivative is nonsingular, then that solution is a point of attraction for Newton's method. The paper [35] proves this result and provides an illustration of the fast convergence on an EMG that is not a traditional variational inequality. We believe this result is significant for applications, because variants of Newton's method are generally considered to be the most effective methods of finding solutions to large classes of nonlinear problems.

2.5 Work in Progress

1. We are developing the use of stochastic programming techniques to facilitate the shaping of dose volume histograms in radiotherapy treatment planning in conjunction with Andrzej Ruszczyński (Rutgers University), Darinka Dentcheva (Stevens Institute of Technology), and a student, Laura Legault. While this has been reported in several talks listed above, the technical report describing the work is still in preparation. However, the work has used the models and data refinement techniques that were described in our paper [23] for general optimization approaches for radiation therapy.

2. We are developing methods for optimizing crop management schemes (including rotation, herd size, feed types and quantities) to move towards sustainable agricultural practices, in collaboration with Professor Thomas L. Cox of the Department of Agricultural and Applied Economics at Madison. This project has ramifications involving energy modeling, specifically when considering bio-fuels. Ferris is continuing to interact with faculty and post-docs in environmental engineering and beyond to facilitate the use of optimization models in this domain.

3. We are continuing to look at applications and solution of large scale models involving complementarity. Specifically, in collaboration with Schmedders (Chicago) and Judd (Hoover, Stanford) we are looking at issues related to approximating equilibria in stochastic overlapping generation models when markets are complete. Collaboration with Roger Wets is considering algorithms for stochastic equilibria when contracts are allowed. A student Holzer is investigating convergence and global solution of algorithms for problems with embedded complementarity structures.

4. We are investigating applications of the work described above involving equations on monotone graphs, to various areas including the extended nonlinear programming models of Rockafellar, the computation of robust statistical estimators, and other areas.

3 Personnel supported

The following professional personnel received salary support from the grant during this reporting period:

- Michael C. Ferris, Professor
- Jesse T. Holzer, Research Assistant
- Shu Lu, Research Assistant
- Andrew J. Miller, Assistant Professor
- Stephen M. Robinson, Professor (2007), Professor Emeritus (2008–)

4 Publications

This section reports the status of publications supported by this grant. It has five subsections, of which the first reports a book and the second reports the status of some papers sponsored by the predecessor AFOSR grant (FA9550-04-1-0192) for which the publication process was not complete at the time the final report for that grant was submitted. The third and fourth report papers (published, accepted, or submitted) and technical reports, respectively, supported by this grant; some of these were also supported by the predecessor grant. The final subsection lists four Ph.D. dissertations, two of which (Deng and Lu) were supported in part by predecessor AFOSR funding and completed during the term of this grant.

4.1 Book

1. M. C. Ferris, O. L. Mangasarian, and S. J. Wright. *Linear Programming with MATLAB*. Number 7 in MPS-SIAM Series on Optimization. SIAM, Philadelphia, PA, 2007.

4.2 Papers sponsored by predecessor grant

2. G. Deng and M. C. Ferris. Extension of the DIRECT optimization algorithm for noisy functions. In B. Biller, S. Henderson, M.-H. Hsieh, J. Shortle, J. Tew, and R. R. Barton, editors, *Proceedings of the 2007 Winter Simulation Conference*, pages 497–504, 2007.
3. M. C. Ferris, A. J. Wathen, and P. Armand. Limited memory solution of bound constrained convex quadratic problems arising in video games. *RAIRO Operations Research*, 40:19–34, 2007.

4. J.-H. Lim, M. C. Ferris, S. J. Wright, D. M. Shepard, and M. A. Earl. An optimization framework for conformal radiation treatment planning. *INFORMS Journal on Computing*, 19:366–380, 2007.

5. Y. Guan and A. J. Miller. Polynomial time algorithms for stochastic uncapacitated lot-sizing problems. *Operations Research*, 56:1172–1183, 2008.

6. Y. Guan and A. J. Miller. A polynomial time algorithm for the stochastic uncapacitated lot-sizing problem with backlogging. In *Proceedings of IPCO, Bertinoro, Italy, May 2008*, volume 5035 of *Lecture Notes in Computer Science*, pages 450–462. Springer-Verlag, 2008.

7. Stephen M. Robinson and Shu Lu. Solution continuity in variational conditions. *Journal of Global Optimization*, 40:405–415, 2008.

8. Shu Lu. Sensitivity of static traffic equilibria with perturbations in arc cost function and travel demand. *Transportation Science*, 42:105–123, 2008.

9. Jenna L. Marquard and Stephen M. Robinson. Reducing perceptual challenges in making decisions with models. In T. Kugler, J. C. Smith, T. Connolly, and Y.-J. Son, editors, *Decision Modeling and Behavior in Complex and Uncertain Environments*, pages 33–55. Springer, New York, 2008. ISBN 978-0-387-77130-4.

10. G. Deng and M. C. Ferris. Variable-number sample-path optimization. *Mathematical Programming*, 117:81–109, 2009.

4.3 Papers sponsored by this grant

4.3.1 Appeared

11. X. Ban, H. X. Liu, M. C. Ferris, and B. Ran. A link-node complementarity model and solution algorithm for dynamic user equilibria with exact flow propagations. *Transportation Research Part B*, 42(9):823–842, 2008.

12. P. Prakash, G. Deng, M. C. Converse, J. G. Webster, D. M. Mahvi, and M. C. Ferris. Design optimization of a robust sleeve antenna for hepatic microwave ablation. *Physics in Medicine and Biology*, 53:1057–1069, 2008.

13. E. Bartholomew Fisher, R. O'Neill, and M. C. Ferris. Optimal transmission switching. *IEEE Transactions on Power Systems*, 23:1346–1355, 2008.

14. M. C. Ferris, C. T. Maravelias, and A. Sundaramoorthy. Using grid computing to solve hard planning and scheduling problems. In *Proceedings of 18th European Symposium on Computer-Aided Process Engineering (ESCAPE 18)*, Lyon, France, June 1-4 2008.

15. M. C. Ferris, S. P. Dirkse, J.-H. Jagla, and A. Meeraus. Extending modeling systems: Structure and solution. In M. Ierapetriou, M. Bassett, and S. Pistikopoulos, editors, *Proceedings of the Fifth International Conference on Foundations of Computer-Aided Process Operations (FOCAPO 2008)*, Cambridge, MA, 2008. Omni Press.

16. M. C. Ferris, S. P. Dirkse, J.-H. Jagla, and A. Meeraus. An extended mathematical programming framework. *Computers and Chemical Engineering*, 33:1973–1982, 2009.

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23. Stephen M. Robinson. Solution continuity in monotone affine variational inequalities. *SIAM Journal on Optimization*, 18:1046–1060, 2007.

4.3.2 Accepted

24. B. Denton, A. J. Miller, H. Balasubramanian, and T. Huschka. Optimal allocation of surgery blocks to operating rooms under uncertainty. Accepted by *Operations Research.*

25. X. Ban, M. C. Ferris, and H. X. Liu. Numerical studies on reformulation techniques for continuous network design with asymmetric user equilibrium. *International Journal of Operations Research and Information Systems*, forthcoming, 2010.

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4.3.3 Submitted

29. X. Ban, M. C. Ferris, and L. Tang. Risk neutral second best toll pricing. Technical Report RPI-WP-200902, Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, `http://www.rpi.edu/~banx/publications/RNSBTP.pdf` (Accessed: 01/02/2009), 2009. Submitted to *Transportation Research Part B.*

30. A. D. A. Gunawardena, M. C. Ferris, and R. R. Meyer. A network approach for intensity modulated arc therapy. Technical report, University of Wisconsin, Madison, Wisconsin, 2009. Submitted to *Optimization Methods and Software.*

31. Q. Li, M. C. Ferris, and T. S. Munson. Linear algebra enhancements to the PATH Solver. Technical Report, University of Wisconsin, Madison, Wisconsin, 2009. Submitted to *Transactions on Mathematical Software.*

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4.4 Technical Reports

1. X. Ban, S. Lu, M.C. Ferris, and H. Liu. Considering risk-taking in second best toll pricing. Technical Report RPI-WP-200901, Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, `http://www.rpi.edu/~banx/publications/RASBTP.pdf` (Accessed: 01/02/2009), 2009.

2. G. Deng, P. Prakash, M. C. Converse, J. G. Webster, and M. C. Ferris. A two-phase optimization framework for designing coaxial antennas for hepatic microwave ablation. Technical report, Computer Sciences Department, University of Wisconsin, 2007.

4.5 Students graduated with dissertation

1. Qian Li, *Large-Scale Computing for Complementarity and Variational Inequalities,* Ph.D. Dissertation, Computer Sciences Department, University of Wisconsin-Madison, January 2010.

2. Laura Legault, *An Interactive GUI Tool for Radiotherapy Treatment Planning,* M.S. Thesis, Computer Sciences Department, University of Wisconsin-Madison, May 2009.

3. Shu Lu, *Sensitivity of Variational Inequalities over Perturbed Polyhedral Convex Sets: Analysis and Implementation,* Ph.D. Dissertation, Department of Industrial and Systems Engineering, University of Wisconsin-Madison, May 2007.

4. Geng Deng, *Simulation Based Optimization,* Ph.D. Dissertation, Computer Sciences Department, University of Wisconsin-Madison, August 2007 (not reported in previous final report).

5 Interactions

This section reports meeting presentations or participations (Section 5.1), along with additional professional interactions (Section 5.2), as well as consultative and advisory functions (Section 5.3).

5.1 Participation/presentations at meetings, conferences, seminars, etc.

Lectures by Ferris are available in PDF format at `http://www.cs.wisc.edu/~ferris`. Those by Robinson are available by request to `smrobins@wisc.edu`.

1. Stephen M. Robinson, "Solution Continuity in Affine Variational Inequalities," Technische Universität München, Germany, June 2007
2. Stephen M. Robinson, "Affine Variational Inequalities: Structure and Applications," Distinguished Lecture Series, Department of Industrial and Enterprise Systems Engineering, University of Illinois at Urbana-Champaign, September 2007
3. Stephen M. Robinson, "Some Continuity Properties of Affine Variational Inequalities," INFORMS Annual Meeting, Seattle, WA, November 2007
4. Jenna L. Marquard and Stephen M. Robinson, "Making Decisions with Models: Uncovering Perceptual and Cognitive Challenges," INFORMS Annual Meeting, Seattle, WA, November 2007
5. Michael C. Ferris, "Optimization in Radiotherapy: A Review", Tomotherapy, Madison, January 2008
6. Michael C. Ferris, "Nonlinear Complementarity Problems and Extensions", INFORMS Practice Meeting, Baltimore, April 2008
7. Michael C. Ferris, "Embedded Mathematical Programming: Structure and Solution", SIAM Optimization Meeting, Boston, May 2008
8. Stephen M. Robinson, "Reparametrization Methods for Variational Problems," Conference on Nonlinear Analysis and Optimization, Technion–Israel Institute of Technology, Haifa, Israel, June 2008

9. Michael C. Ferris, "Embedded Mathematical Programming: Structure and Solution", Foundations of Computer-Aided Process Operations (FOCAPO), Boston, July 2008

10. Michael C. Ferris, "Optimization Tools in an Uncertain Environment", Workshop on Modeling Uncertainty in Integrated Assessment Models, Chicago, July 2008

11. Michael C. Ferris, "Radiation Treatment Planning: A View from Optimization", American Association of Physicists in Medicine National Meeting, Houston, July 2008

12. Michael C. Ferris, "Optimization with Medicine and Biology", UW Symposium on Integration of Mathematical and Biological Sciences, Madison, September 2008

13. Michael C. Ferris, "Optimization of Noisy Functions", INFORMS National Meeting, Washington, DC, October 2008

14. Stephen M. Robinson, "Local Analysis of Functions on Thin Graphs," Fifth Rutgers-Stevens Workshop, Rutgers University, New Brunswick, NJ, March 2009

15. Michael C. Ferris, "Optimization Modeling: Recent Enhancements and Future Extensions", Invited Plenary, SIAM National Meeting, Denver, July 6 2009. Video taped capture of the talk available from SIAM website.

16. Stephen M. Robinson, "Functions on Monotone Graphs: Analysis and Computation," International Symposium on Mathematical Programming, Chicago, IL, August 2009

17. Michael C. Ferris, "An Extended Mathematical Programming Framework", University of California, Berkeley, April 27 2009; INFORMS Computer Science Meeting, Charleston, January 11 2009; International Symposium on Mathematical Programming, Chicago, IL, August 26, 2009; University of California at Davis, January 15, 2010

18. Michael C. Ferris, "Facebook friend wheels and quadratic assignment problems," INFORMS Annual Meeting, San Diego, CA, October 11, 2009

19. Stephen M. Robinson, “Some Issues in Stochastic Variational Problems,” International Colloquium on Stochastic Modeling and Optimization, Rutgers University, New Brunswick, NJ, November 2009

20. Michael C. Ferris, “Optimization in Biology and Medicine: Wisconsin Institutes of Discovery”, Architecture Affiliates Meeting, October 8, 2009 ; Computer Sciences Board of Visitors Meeting, October 16, 2009; ISyE Colloquium, February 5, 2010

21. Michael C. Ferris, “New Stochastic Programming Approaches for Radiation Therapy Planning”, Multi Criteria Optimization in Radiation Therapy, Massachusetts General Hospital, Boston, October 23, 2009; Lehigh University, November 17, 2009 University of Michigan, December 2, 2009

5.2 Other professional service

Michael C. Ferris:

1. Chair, SIAM Activity Group on Optimization, (2008 – 2010).
2. Member, ICCOPT Steering Committee, Mathematical Programming Society, (2008 – 2009).
3. Program Committee, SIOPT ’08, Boston, MA.
4. Organizing Committee, International Symposium on Mathematical Programming 2009, Chicago, IL: in charge of commercial sponsorship and vendors.
5. Program Committee, International Symposium on Mathematical Programming 2009, Chicago, IL.
6. INFORMS Nicholson Prize Committee 2008–2009, (Chair in 2009). Reviewed 45 papers each year for prize, attended prize talks.

Stephen M. Robinson:

1. Since January 2007 Robinson has served as Treasurer of the Institute for Operations Research and the Management Sciences (INFORMS) and concurrently as a member of its Executive Committee.

5.3 Consultative and advisory functions

In 2008-9 Robinson served as a member of the Committee on Experimentation and Rapid Prototyping in Support of Counterterrorism of the National Research Council (NRC). The committee was chaired by Dr. Paul G. Kaminski, former Under Secretary of Defense for Acquisition and Technology. It conducted a study of current DOD approaches to rapid prototyping and published a report, *Experimentation and Rapid Prototyping in Support of Counterterrorism* (The National Academies Press, Washington, DC 2009, 91 pp., ISBN-13: 978-0-309-13668-6).

Robinson has also served since 2009 as a member of the NRC review panel for the Survivability and Lethality Analysis Division (SLAD) of the U. S. Army Research Laboratory (ARL) . This panel conducts annual on-site technical and organizational assessments of SLAD, and makes recommendations to assist ARL in its operations.

At the invitation of the U. S. Army Research Office (ARO), Robinson attended the Decision Sciences Workshop organized at the United States Military Academy (USMA), West Point, NY during 16-17 June 2009, jointly sponsored by ARO and USMA, and gave a presentation on "Analytic Support to Irregular Warfare Operations."

6 New discoveries, inventions, or patent disclosures

There were none during the period of this grant.

7 Honors/awards

Honors and awards earned by the principal investigators are reported in two categories: those received during the period of the current grant, and lifetime achievement honors previously received.

7.1 Period of current grant

- Elected member of the National Academy of Engineering, 2008 (Robinson)
- Life appointment as National Associate of the National Research Council, 2008 (Robinson)

- Fellow of the Society for Industrial and Applied Mathematics, 2009 (Robinson)

7.2 Lifetime achievement honors not reported above

- Doctor *honoris causa,* University of Zürich, Switzerland, 1996 (Robinson)
- George B. Dantzig Prize, Mathematical Programming Society and the Society for Industrial and Applied Mathematics, 1997 (Robinson)
- Beale-Orchard-Hays Prize, Mathematical Programming Society, 1997 (Ferris)
- Fellow of the Institute for Operations Research and the Management Sciences, 2004 (Robinson)
- Fellow of the Institute for Operations Research and the Management Sciences, 2006 (Ferris)

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